The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 7, 2010 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and homework solution sets.
- **Power off all cell phones**
- You may use any standalone calculator or other computing system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	18		Differential Equation
2	18		Convolution
3	24		System Properties
4	28		Equalization
5	12		Potpourri
Total	100		

Problem 1.1 Differential Equation. *18 points*.

For a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 6y(t) = x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? *3 points*.

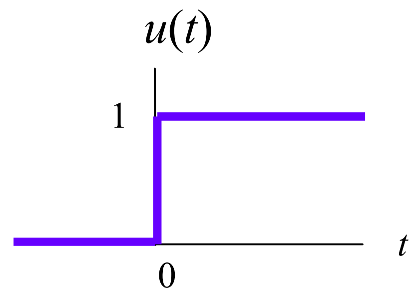
(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. *6 points*.

(c) Find the zero-input response for the following initial conditions: $y(0^+) = 0$ and $y'(0^+) = 1$. *6 points*.

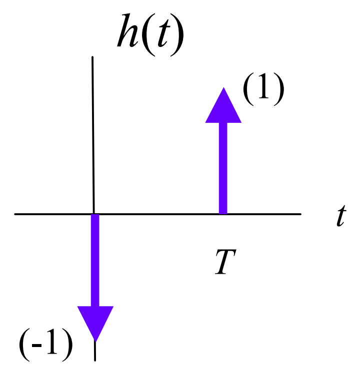
(d) Is the zero-input response asymptotically stable, marginally stable, or unstable? Why? *3 points*.

Problem 1.2 Convolution. *18 points*.

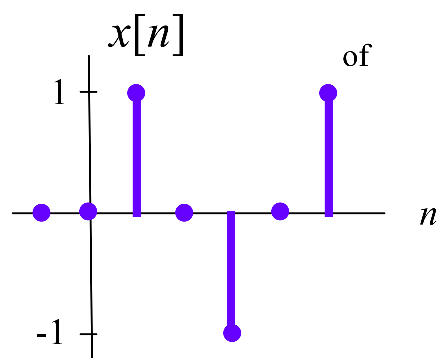
Sketch (plot) the following convolutions. On the sketches, be sure to label significant points on the horizontal and vertical axes. You do not have to show intermediate work, but showing intermediate work may qualify for partial credit.

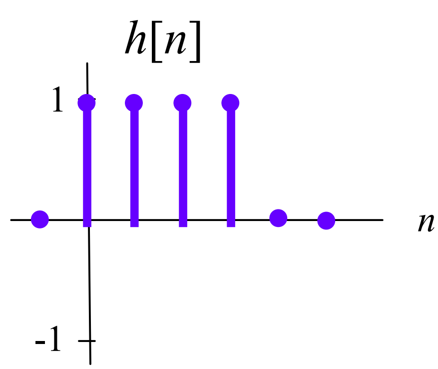


(a) In continuous-time, convolve the unit step function $u(t)$ with $h(t)$ where $h(t) = -\delta(t) + \delta(t - T)$. Both signals are plotted on the right. *9 points*



(b) In discrete time, convolve a one-sided sine wave $x[n]$ where $x[n] = \sin(\frac{\pi}{2} n)\, u[n]$ with a rectangular pulse of four samples in duration $h[n]$. Both signals are plotted on the right for $n \in [-1, 5]$. *9 points*





Problem 1.3 System Properties. *24 points*.

Consider a continuous-time quantizer with input $x(t)$ and output $y(t)$ where

$$y(t) = \begin{cases} +1 & \text{if } x(t) \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

The quantizer is a pointwise operation; that is, the current output value depends only on the current input value.

Either prove each of the following statements to be true, or give a counterexample to show that the statement is false. *Please note that writing only true or false will receive zero points.*

(a) The system is linear. *6 points.*

(b) The system is time-invariant. *6 points.*

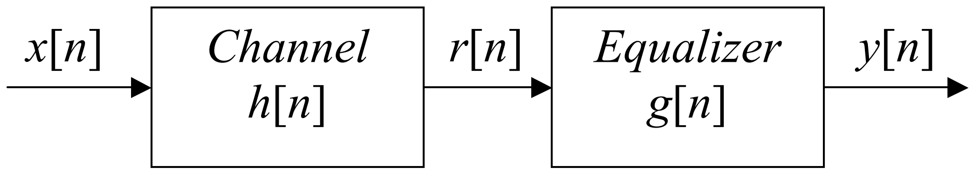
(c) The system is causal. *6 points.*

(d) The system is memoryless. *6 points.*

Problem 1.4 Equalization. *28 points*.

In a communication system, the receiver uses equalization to compensate for distortion that a transmitted signal experiences when propagating through the communication channel.

Consider the following discrete-time model of a communication system:



$g[n]$ is the impulse response of a linear time-invariant equalizer,
$h[n]$ is the impulse response of a linear time-invariant communication channel,
$r[n]$ is the received signal,
$x[n]$ is the transmitted signal, and
$y[n]$ is the received signal.

The equalizer should be designed so that $h[n] * g[n] = \delta[n]$ in order to make $y[n] = x[n]$.

(a) Let $h[n] = (\frac{1}{2})^n\, u[n]$ and $g[n] = g_0\, \delta[n] + g_1\, \delta[n-1]$. What are the values of g_0 and g_1? *14 points*.

(b) Let $h[n] = h_0\, \delta[n] + h_1\, \delta[n-1]$. Give a formula for $g[n]$ in terms of h_0 and h_1. *14 points*.

Problem 1.5 Potpourri. *12 points*.

Give one signal processing or communication system that uses each of the following subsystems and describe the role that the subsystem plays in the overall system. You can answer the question in either continuous time or discrete time.

(a) Linear time-invariant subsystem that is bounded-input bounded-output stable. *6 points*.

(b) Linear time-invariant subsystem that is bounded-input bounded-output unstable. *6 points*.